Learning Optimal "Pigovian Tax" in Sequential Social Dilemmas

Yun Hua^{*} Shang Gao[†] Wenhao Li[‡] Bo Jin[§] Xiangfeng Wang[¶] Hongyuan Zha^{II}

Abstract

In multi-agent reinforcement learning, each agent acts to maximize its individual accumulated rewards. Nevertheless, individual accumulated rewards could not fully reflect how others perceive them, resulting in selfish behaviors that undermine global performance. The externality theory, defined as "the activities of one economic actor affect the activities of another in ways that are not reflected in market transactions," is applicable to analyze the social dilemmas in MARL. One of its most profound non-market solutions, "Pigovian Tax", which internalizes externalities by taxing those who create negative externalities and subsidizing those who create positive externalities, could aid in developing a mechanism to resolve MARL's social dilemmas. The purpose of this paper is to apply externality theory to analyze social dilemmas in MARL. To internalize the externalities in MARL, the Learning Optimal Pigovian Tax method (LOPT), is proposed, where an additional agent is introduced to learn the tax/allowance allocation policy so as to approximate the optimal "Pigovian Tax" which accurately reflects the externalities for all agents. Furthermore, a reward shaping mechanism based on the approximated optimal "Pigovian Tax" is applied to reduce the social cost of each agent and tries to alleviate the social dilemmas. Compared with existing state-of-the-art methods, the proposed LOPT leads to higher collective social welfare in both the Escape Room and the Cleanup environments, which shows the superiority of our method in solving social dilemmas.

1 Introduction

Reinforcement Learninghas achieved wide success in various scenarios [21, 16, 13, 39] and has been successfully expanded into the multi-agent setting, especially in fully-cooperative games [33, 19, 36]. However, most multi-agent reinforcement learning (MARL) methods that use centralized learning methods with a team reward [8, 28, 26, 25] are excluded as they don't scale feasibly to large populations and are not suitable for self-interested agents. In addition, decentralized learning methods [29, 27, 2], where agents are designed to maximize their individual rewards based on their personal interests, have difficulty dealing with coordination among agents. In many real-world environments with mixed-motives, such as those within exclusionary and subtractive common-pool resources [24, 17, 18], selfish agents may fall into social dilemmas because of the temptation to evade any cost, which brings extra social costs and negative influences on social welfare.

Social dilemma originates from economics and describes situations in which individual rationality leads to collective irrationality [14], where a more precise definition is that everyone benefits from mutual cooperation but individuals profit disproportionately from non-cooperative behaviors. Similarly, in MARL, it

^{*}School of Computer Science and Technology, East China Normal University, Shanghai 200092, China. (E-mail: yun-hua@stu.ecnu.edu.com)

[†]School of Computer Science and Technology, East China Normal University, Shanghai 200092, China. (E-mail: shang-gao@stu.ecnu.edu.com)

[‡]School of Data Science, The Chinese University of Hong Kong, Shenzhen, Shenzhen Institute of Artificial Intelligence and Robotics for Society, Shenzhen, Guangdong, China. (E-mail: liwenhao@cuhk.edu.cn)

[§]School of Software Engineering, Tongji University, Shanghai 201804, China. (E-mail: bjin@tongji.edu.cn)

[¶]School of Computer Science and Technology, East China Normal University, Shanghai 200062, China. (E-mail: xfwang@sei.ecnu.edu.cn)

⁸School of Data Science, The Chinese University of Hong Kong, Shenzhen, Shenzhen Institute of Artificial Intelligence and Robotics for Society, Shenzhen, Guangdong, China. (E-mail: zhahy@cuhk.edu.cn)

is defined as a conflict between agents' self-interest and team reward [17]. Previous works in economics try to apply the *externality theory* in dealing with social dilemmas [30], where an externality is proposed to present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent [20]. Therefore, the externality theory is a practical tool to measure the influence on social welfare caused by self-interested agents. Positive externality arises from agents who benefit social welfare, while negative externality comes from agents who harm it. Various non-market and market solutions have been proposed to reduce the negative influence [23, 1, 4]. "Pigovian Tax" is one of the most popular solutions [4] in non-market economics, which levies taxes on any market activity that generates negative externalities and provides allowances to market activities that bring positive externalities [22]. In this instance, the Carbon Tax is an example of a Pigovian tax that reveals the "hidden" social costs of carbon emissions. Indirectly, these costs are felt through more severe weather events.

It is therefore natural to introduce externality theory to the social dilemmas in MARL, which can provide a theoretical foundation to explain its emergence. In these cases, the "Pigovian Tax" will offer a potential solution to the problem of social dilemmas by addressing externality. Consequently, our main concern is how to measure externality and develop an effective tax/allowance mechanism to reduce negative externality and promote positive externality.

In this paper, we model externality as measuring social dilemmas with theoretical justifications. Afterward, a Pigovian tax/allowance mechanism is proposed to alleviate social dilemmas by discouraging negative externalities while encouraging positive externalities. The proposed method Learning **O**ptimal **P**igovian **T**ax (**LOPT**) employs a centralized agent, called the **tax planner**, which is built to learn the Pigovian tax/allowance mechanism and allocate tax/allowance rates based on the team reward. Learning the tax/allowance allocation policy maximizes the long-term team reward, which is equivalent to approximating the optimal Pigovian tax. According to the learned tax/allowance rates, a distinctive-structure reward shaping, *optimal Pigovian tax reward shaping*, is established. Different from recent reward shaping structures, which are either hand-crafted or evolved based on other agents' performance [7, 10, 34]. The proposed LOPT is based on the approximated optimal Pigovian tax within a percentage tax/allowance formulation. As a result of this reward structure, each agent's externality is visualized so that its influence on the environment can be quantified.

The primary contributions of this paper are as follows: 1). Externality theory is first introduced to measure the influence of each agent's policy on social welfare and support to analyze of the social dilemma problems in MARL; 2). A centralized tax/allowance mechanism based on reward shaping is proposed to get the optimal Pigovian tax and solve the externality in MARL tasks; 3). Experiments in the Escape Room environment and the challenging Cleanup environment ¹ show the superiority of the mechanism for alleviating social dilemmas in MARL.

2 Related Works

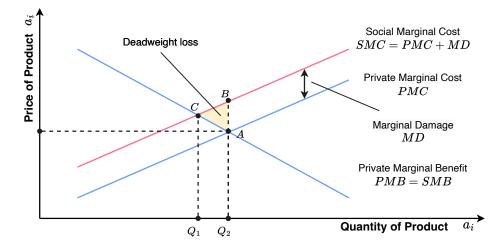
LOPT is motivated by the problem of cooperation among independent learning agents within intertemporal social dilemmas (ISDs) [17]. In such a problem, agents aim to maximize their individual accumulated rewards, but mutual defection leads to low social welfare in the long term. MARL algorithms designed for fully-cooperative tasks [8, 28, 26] are incapable of ISDs, as in such problems, agents have mixed motivations. Our work is inspired by the externality theory [20] in economics, which focuses on the well-being of a consumer or the production possibilities of a firm that are directly affected by the action of another entity. Externality theory provides a theoretical framework to analyze how each entity in economic society affects social welfare, which is highly related to intertemporal social dilemmas. Moreover, in economics, various solutions in both non-market economies [1] and market economies [23] have been proposed to solve the externality problem and improve social welfare. LOPT is based on the most popular non-market economy solution, the Pigovian tax [4]. Pigovian tax is the tax on any market activity that generates negative ex-

¹Code for our implementation is available at https://github.com/Felix2048/LOPT.

ternalities so that the hidden social influence becomes visible to the entity in the economic society and the externality is internalized. Additionally, some works in economics, like AI-Economist [38], use reinforcement learning to improve tax policies. LOPT uses a similar two-stage tax planner structure to AI Economist. However, LOPT use externality theory and Pigovian tax/allowance to deal with the social dilemma problems in the MARL area.

In the ISDs, optimal Pigovian tax reward shaping assigns negative reward shaping to any behavior that does harm the social welfare and positive reward shaping to behaviors that promote social welfare. So, the self-interested agent can obtain adequate feedback from the global influence to solve the ISDs problems. Most reward shaping or intrinsic reward-based methods [7, 10, 34] dealing with ISDs are either hand-crafted or evolved based on other agents' performance. The recent work LIO [37] learns the 'incentivise reward' give to other agents. Our proposed LOPT learns a novel reward shaping based on externality theory, implemented with a centralized agent with global information.

Besides, our method concerns the structural solutions for ISDs problems. Recent works consider two main solutions, the centralized boundaries [5, 11] and the decentralized sanctions [3, 15, 35, 32, 6], which are highly similar to the non-market and market economics solutions to the externality. The centralized boundaries aim to build an external authority like the government in an economic society to regularize agents' behavior. The decentralized sanctions aim to establish a self-protection mechanism so that each agent can punish other agents who harm social welfare. Our work is a centralized boundaries solution, as we build a centralized tax planner to make the Pigovian tax policy. For previous works in the centralized boundaries, [5] introduces a global shared arbitrary and polls agents to the limited common resource as the ISDs problems are highly related to the shared resource allocation, and [11] brings the tax mechanism to solve the ISDs problems. These methods need to design specific policies for different environments without learning. By contrast, our method learns the tax policy in different environments and theoretically adopts the externality to support the learning process.



3 Externality in MARL

Figure 1: The Externality [20]. The gap between social marginal cost and the private cost is the externality.

This section illustrates the externality in MARL and builds a formulation to measure the externality, making it possible to visualize social dilemmas. First, the externality in economics is explained, with the graphical analysis shown in Figure 1. Let us suppose a firm i produces some product a_i to satisfy the consumer. At the same time, the firm also produces pollution, which harms social welfare. We might guess that the quantity of the produced a_i is q_i , and the price of a_i is a function based on the quantity of the

produced a_i and the market requirement of a_i . Define the market requirement as q_i^r , and the price function is denoted as $P_i(q_i^r, q_i)$. The target of the firm is to maximize such utility:

$$u_i\left(q_i, q_i^r\right) = q_i \times P_i\left(q_i^r, q_i\right). \tag{1}$$

Consider N firms indexed i = 1, 2, ..., N, and each produces some product a_i . Naturally, each firm aims to maximize its profit. So all of them follow the Equation. 1. However, activities that are not reflected in market transactions, e.g., pollution, which harms social welfare or creates jobs that benefit social welfare, must occur within the production process. Essentially, social welfare needs to consider these activities. In this way, we define the influence of such activities of each firm i as a function $x_i(q_i)$ based on the quantity of the produced a_i . Then, social welfare is acquired:

$$U = \sum_{i} u_i \left(q_i, q_i^r \right). \tag{2}$$

The externality is caused by these activities which are not reflected in market transactions with economical definition as:

Definition 1. An *externality* occurs whenever one economic actor's activities affect another's activities in ways that are not reflected in market transactions [20].

The influence x_i can be used to measure the externality. When $x_i > 0$, it is the positive externality. While $x_i < 0$, it is the negative externality. We can use a function $t_i(q_i)$ based on the quantity of the produced a_i to express the Pigovian tax, then the after-tax utility for firm i is:

$$u_{i}(q_{i}, q_{i}^{r}) = q_{i} \times P_{i}(q_{i}^{r}, q_{i}) - t_{i}(q_{i}).$$
(3)

It shows that the value of Pigovian tax is equivalent to the influence x_i in such formulation. If we can acquire the exact influence x_i of the firm *i*, the optimal Pigovian tax will be accessed and succeed in internalizing the externality. Since the externality is also used to express an agent's influence on social welfare, we expand the definition to the MARL area:

Definition 2. An *Externalities* occurs whenever an agent's actions affect others in ways that are not reflected in local rewards.

Eventually, Definition .1 makes the most basic definition for externality. We use Definition .2 to follow the basic definition and assume the market has a constant outside opportunity cost, and directly calculate aggregate externalities, which homogenize the influence of different agents.

A decentralized MARL scenario is considered with a N-player partially observable general-sum Markov game on a finite set of states S. In each timestep, agents receive their d-dimensional views from the observation function $\mathcal{O} : s \times \{1, \dots, N\} \to \mathbb{R}^d$ based on the current state $s \in S$. Then, agents select action $\{a_i\}_{i=1}^N \in \{\pi_i(a \mid o_i)\}_{i=1}^N$ from the set of actions $\{\mathcal{A}\}_{i=1}^N$, which transfers to the next state s' according to the transition function $P(s \mid \{a_i\}_{i=1}^N)$. And agents get their individual extrinsic rewards $\{r_i = \mathcal{R}_i(s, \mathbf{a})\}$. Each agent aims to maximize its long-term γ -discounted payoff:

$$Q^{i}(s, \mathbf{a}) = \mathbf{E}\left[\sum_{t=0}^{T} \gamma^{t} r_{i}(s^{t}, \mathbf{a}^{t}) \mid s^{0} = s, \mathbf{a}^{0} = \mathbf{a}\right].$$
(4)

The social welfare of the scenario is defined as a global long-term γ -discount payoff as follows:

$$Q(s, \mathbf{a}, \mathbf{x}) = \mathbb{E}\left[\sum_{t=0}^{T} \gamma^{t} \sum_{i=1}^{N} r_{i}\left(s^{t}, \mathbf{a}^{t}\right) + x_{i}\left(s^{t}, a_{i}^{t}\right) \left|s^{0} = s, \mathbf{a}^{0} = \mathbf{a}\right],\tag{5}$$

where the $x_i(s^t, a_i^t)$ shows the influence of agent *i* on other agents in the scenario, and **x** is the joint influence $\{x_i\}_{i=1}^N$. Nevertheless, in such a setting, each agent's behavior must influence the rewards of other agents. As a result, social welfare is equivalent to:

$$Q(s, \mathbf{a}) = \mathbb{E}\left[\sum_{t=0}^{T} \gamma^t \sum_{i=1}^{N} r_i\left(s^t, \mathbf{a}^t\right) \mid s^0 = s, \mathbf{a}^0 = \mathbf{a}\right].$$

The optimal joint policy leads to the following social welfare:

$$Q^*(s, \mathbf{a}^*) = \mathbb{E}\left[\sum_{t=0}^T \gamma^t \sum_{i=1}^N r_i\left(s^t, \mathbf{a}^t\right) \mid s^0 = s, \mathbf{a}^0 = \mathbf{a}^*\right],$$

where \mathbf{a}^* is the optimal joint action from the optimal joint policy. From Definition 2, the externality of the agent *i* can be defined as follows:

$$E^{i}(s, \mathbf{a}_{-i}^{*}, a_{i}) = Q^{*}(s, \mathbf{a}^{*}) - Q(s, \mathbf{a}_{-i}^{*}, a_{i}),$$
(6)

where \mathbf{a}_{-i} is the joint action without a_i and a_i is the current action of agent *i*. From the (1), an Optimal Pigovian Tax reward shaping can be proposed to solve the externality in MARL to solve the social dilemmas. The optimal Pigovian tax based reward shaping can be written as follows:

$$F_i(s, \mathbf{a}_{-i}^*, a_i) = Q^*(s, \mathbf{a}^*) - Q(s, \mathbf{a}_{-i}^*, a_i).$$
⁽⁷⁾

The agent *i* receives a reward with the reward shaping:

$$\hat{r}_i\left(s^t, \mathbf{a}^t\right) = r_i\left(s^t, \mathbf{a}^t\right) + F_i\left(s, \mathbf{a}_{-i}^*, a_i\right),\tag{8}$$

which will succeed in internalizing the externality.

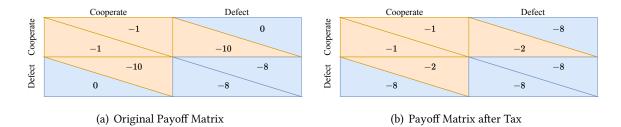


Figure 2: Pigovian Tax/Allowance for Prisoner's Dilemma.

A typical example called "Prisoner's Dilemma" is proposed in Figure 2, where the two captured prisoners must choose to cooperate or defeat. From the original payoff matrix shown in Figure 2(a), although both of them choosing to cooperate will lead to the best results, choosing to defeat is the dominant action based on each agent's self-interest. Therefore, these two agents will drop into the result with the lowest social welfare. Eventually, the Prisoner's Dilemma is caused by the difference between private and social costs. Thus, the externality is naturally suitable to describe such a situation as each agent does not have access to the negative externality from their actions. This way, we calculate their externality by (6) and then get the Optimal Pigovian Tax reward shaping by (7). Thus, the new payoff matrix after tax shown in Figure 2(b) is received. From the payoff matrix after tax, the dominant action based on each agent's self-interest becomes "Cooperate." By internalizing the externality within Optimal Pigovian Tax reward shaping, the social dilemma in "Prisoner's Dilemma" can be solved.

4 Learning Optimal Pigovian Tax

In this section, LOPT will be introduced in detail. As illustrated in Figure 3, it contains two major components: 1) a centralized agent called *Tax Planner* is proposed to allocate the Pigovian tax/allowance for each agent within a functional percentage formulation; 2) a reward shaping structure based on the learned tax/allowance allocation policy is established to make each agent's social cost visible, and further to alleviate the social dilemmas. LOPT is proposed to learn the Optimal Pigovian Tax reward shaping in (7) to make each agent's social cost visible. The Pigovian tax rewards can be reshaped as follows:

$$F_*^i\left(s^t, \mathbf{a}_{-i}^t^*, a_i^t\right) = \sum_{j=0}^N r_j\left(s^t, \mathbf{a}^{t*}\right) - \sum_{j=0}^N r^j\left(s^t, \mathbf{a}_{-i}^{t*}, a_i^t\right).$$

Pigovian tax reward shaping within percentage tax/allowance is formulated as:

$$F_{\theta,\delta}^{i}\left(s^{t}, \mathbf{a}_{-i}^{t}^{*}, a_{i}^{t}\right) = -\theta_{i}r_{i}\left(s^{t}, \mathbf{a}_{-i}^{t}^{*}, a_{i}^{t}\right) + \delta_{i}(s^{t}, \mathbf{a}^{t})\sum_{j=0}^{N}\theta_{j}r_{j}\left(s^{t}, \mathbf{a}_{-i}^{t}^{*}, a_{i}^{t}\right),$$

where $\boldsymbol{\theta}$ is the tax rates on all agents, θ_i is the specific tax rate for agent *i*, while $\boldsymbol{\delta}$ is the allowance rates on all agents, δ_i is the specific allowance rate for agent *i*. The Optimal Pigovian Tax reward shaping can be learned by learning $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$, so as to let all $F_{\boldsymbol{\theta},\boldsymbol{\delta}}^i\left(s^t, \mathbf{a}_{-i}^{t^*}, a_i^t\right)$ equal to the $F_*^i\left(s^t, \mathbf{a}_{-i}^{t^*}, a_i^t\right)$. However, as the tax and allowance rates are unequal among different agents within different situations, it is necessary to treat $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$ as a function based on the current joint state and action. So the Pigovian tax reward shaping within percentage tax/allowance is rewritten as:

$$F_{\theta,\delta}^{i}\left(s^{t}, \mathbf{a}_{-i}^{t}^{*}, a_{i}^{t}\right) = -\theta_{i}(s^{t}, \mathbf{a}^{t})r_{i}\left(s^{t}, \mathbf{a}_{-i}^{t}^{*}, a_{i}^{t}\right) + \delta_{i}(s^{t}, \mathbf{a}^{t})\sum_{j=0}^{N}\theta_{j}(s^{t}, \mathbf{a}^{t})r_{j}\left(s^{t}, \mathbf{a}_{-i}^{t}^{*}, a_{i}^{t}\right)$$

Theorem 1. If other agents' actions are treated as part of the environment for any agent *i* at any timestep *t*, there always exists typical $\theta_i(s^t, \mathbf{a}^t)$ and $\delta_i(s^t, \mathbf{a}^t)$ to let the $F^i_{\theta,\delta}(s^t, \mathbf{a}^{t}_{-i}^*, a^t_i)$ equal to the $F^i_*(s^t, \mathbf{a}^{t}_{-i}^*, a^t_i)$.

Proof. For any agent *i* which creates a negative externality at timestep *t*: the agent will not receive any allowance, so the allowance rate function $\delta_i(s^t, a_i^t)$ is equal to 0. And the tax rate can be written as:

$$\theta_i(s^t, a_i^t, {a_{-i}^t}^*) = \frac{E^i(s^t, {a_{-i}^t}^*, a_i^t)}{r_i(s^t, a_i^t, {a_{-i}^t}^*)},\tag{9}$$

$$\theta_i(s^t, a_i^t, a_{-i}^{t^*}) = \frac{Q(s^t, \mathbf{a}^{t^*}) - Q(s^t, a_{-i}^{t^*}, a_i^t)}{r_i(s^t, a_i^t, a_{-i}^{t^*})}$$
(10)

And as the interactive influence from other agents is not considered, other agents' optimal action a_{-i}^{t} can be seen as a part of the environment, and this optimum has a fixed result. Therefore, like the reinforcement learning method with an advantage function, for each agent *i*, the advantage function based on the current joint state and action can also be found in the tax rate, where:

$$Q(s^{t}, \mathbf{a}^{t^{*}}) = A_{i}^{0}(s^{t}, \mathbf{a}^{t}) \times Q(s^{t}, \mathbf{a}^{t}),$$

$$Q(s^{t}, a_{-i}^{t^{*}}, a_{i}^{t}) = A_{i}^{1}(s^{t}, \mathbf{a}^{t}) \times Q(s^{t}, \mathbf{a}^{t}),$$

$$r_{i}(s^{t}, a_{i}^{t}, a_{-i}^{t^{*}}) = A_{i}^{2}(s^{t}, \mathbf{a}^{t}) \times r_{i}(s^{t}, \mathbf{a}^{t}).$$
(11)

Then the tax rate for agent *i* becomes:

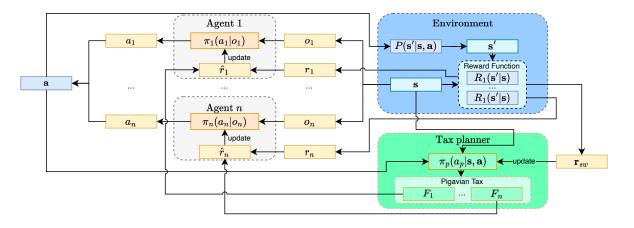
$$\theta_{i}(s^{t}, a_{i}^{t}, a_{-i}^{t}^{*}) = \frac{(A_{i}^{0}(s^{t}, \mathbf{a}^{t}) - A_{i}^{1}(s^{t}, \mathbf{a}^{t})) \times Q(s^{t}, \mathbf{a}^{t})}{A_{i}^{2}(s^{t}, \mathbf{a}^{t}) \times r_{i}(s^{t}, \mathbf{a}^{t})},$$

$$\theta_{i}(s^{t}, \mathbf{a}^{t}) = \frac{(A_{i}^{0}(s^{t}, \mathbf{a}^{t}) - A_{i}^{1}(s^{t}, \mathbf{a}^{t})) \times Q(s^{t}, \mathbf{a}^{t})}{A_{i}^{2}(s^{t}, \mathbf{a}^{t}) \times r_{i}(s^{t}, \mathbf{a}^{t})}.$$
(12)

Then it is proven that for any agent *i* which generates negative externality, there always exists typical $\theta_i(s^t, \mathbf{a}^t)$ and $\delta_i(s, \mathbf{a}^t)$ to let the $F^i_{\boldsymbol{\theta}, \boldsymbol{\delta}}\left(s^t, \mathbf{a}^t_{-i}^*, a^t_i\right)$ equivalent to the $F^i_*\left(s^t, \mathbf{a}^t_{-i}^*, a^t_i\right)$.

Similarly, for any agent *i* which generates positive externality, there also exists typical $\theta_i(s^t, \mathbf{a}^t)$ and $\delta_i(s^t, \mathbf{a}^t)$ to satisfy the condition above.

This proves that if the interactive influence from other agents is not considered, for any agent *i* at any timestep *t*, there always exists typical $\theta_i(s^t, \mathbf{a}^t)$ and $\delta_i(s, \mathbf{a}^t)$ to let the $F^i_{\theta, \delta}(s^t, \mathbf{a}^t_{-i}^*, a^t_i)$ equivalent to the $F^i_*(s^t, \mathbf{a}^t_{-i}^*, a^t_i)$. This theorem shows that the Pigovian tax reward shaping within percentage tax/allowance can reach the optimum in a specific condition. The reward shaping function could be treated as follows:



$$F_{\boldsymbol{\theta},\boldsymbol{\delta}}^{i}\left(s^{t},\mathbf{a}^{t}\right)=F_{\boldsymbol{\theta},\boldsymbol{\delta}}^{i}\left(s^{t},\mathbf{a}_{-i}^{t}^{*},a_{i}^{t}\right)$$

Figure 3: The Architecture of the **LOPT**. The centralized agent Tax planner allocate the Pigovian tax/allowance within a functional percentage formulation. Reward shaping is established based on the Pigovian tax/allowance to alleviate the social dilemmas.

The key issue becomes how to learn the tax and allowance rates function. In Figure 3, a centralized tax planner is built to learn the tax rates and allowance rates functions for Pigovian tax reward shaping. The optimal Pigovian tax based on reward shaping is applied to internalize each agent's externality and solve the social dilemmas. In this form, the tax planner aims to learn the tax rates θ and allowance rates δ for all agents within the MARL task.

Theorem 2. If the interactive influences from other agents are not considered, when the policy of tax planner $\langle \theta_i(s^t, \boldsymbol{a}^t), \delta_i(s^t, \boldsymbol{a}) \rangle$ maximizes the social welfare, the typical $F^i_{\boldsymbol{\theta},\boldsymbol{\delta}}(s^t, \boldsymbol{a}^t_{-i}^*, a^t_i)$ will qualitatively equivalent to the $F^i_*(s^t, \boldsymbol{a}^t_{-i}^*, a^t_i)$.

Proof. Here we use the method of "reduction to absurdity." Suppose that there exists an agent *i* which generates negative externality, and its typically learned $F_{\theta,\delta}^i(s^t, \mathbf{a}_{-i}^t, a_i^t)$ does not qualitatively equivalent to the $F_*^i(s^t, \mathbf{a}_{-i}^t, a_i^t)$. The reason why agent *i* will choose the selfish behavior which harms social welfare without reward shaping is because its individual reward shows:

$$r_i(s^t, a_{-i}^{t^*}, a_i^t) > r_i(s^t, \mathbf{a}^{t^*}).$$
(13)

And the effect of the Optimal Pigovian Tax reward shaping is to let any $a_i^t \in A_i$ hold the following constraint:

$$r_i(s^t, a_{-i}^t, a_i^t) + F_{\theta,\delta}^i(s^t, a_{-i}^t, a_i^t) < r_i(s^t, \mathbf{a}^{t^*}).$$
(14)

As we suppose that its typically learned reward shaping does not qualitatively equivalent to the Optimal Pigovian Tax reward shaping. That means there exists some $a_i^t \in A_i$, which causes:

$$r_i(s^t, a_{-i}^{t^*}, a_i^t) + F_{\theta,\delta}^i(s^t, a_{-i}^{t^*}, a_i^t) > r_i(s^t, \mathbf{a}^{t^*}).$$
(15)

This means agent *i* within its optimal policy π_i^* would like to choose the behavior a_i^t rather than the behavior in optimal joint actions \mathbf{a}^{t^*} . Then if we use the tax planner's learned policy $\pi_p^{\phi_p}$ to describe the tax rate

allocation, which means there exists another tax planner's policy π_p^* , letting:

$$\mathbb{E}_{\pi_p^{\phi_p}}\left[\sum_{t=0}^T r_p\left(s_p^t, a_p^t\right)\right] < \mathbb{E}_{\pi_p^*}\left[\sum_{t=0}^T r_p\left(s_p^t, a_p^t\right)\right].$$
(16)

Thus we have shown that if any learned reward shaping of agent i is not qualitatively equivalent to the Optimal Pigovian Tax reward shaping, the tax planner's learned policy is not optimal.

This proves that if the interactive influence from other agents is considered, when the policy of tax planner $\langle \theta_i \left(s^t, \mathbf{a}^t \right), \delta_i \left(s^t, \mathbf{a}^t \right) \rangle$ maximizes the social welfare, the typical $F^i_{\theta,\delta} \left(s^t, \mathbf{a}^t_{-i}^*, a^t_i \right)$ will qualitatively equivalent to the $F^i_* \left(s^t, \mathbf{a}^{t-i}_{-i}^*, a^t_i \right)$.

This theorem shows that treating the tax planner as a reinforcement learning agent and letting it maximize social welfare will approximate the "optimal Pigovian tax" based reward shaping. Therefore, the tax planner can be defined as a centralized reinforcement learning agent as follow: $\langle S_p, \mathcal{O}_p, \mathcal{A}_p, \mathcal{R}_p \rangle$, where S_p is the global state space, and \mathcal{O}_p is the observation function to get observation o_p from the global state, \mathcal{A}_p is the action space for the tax planner, and \mathcal{R}_p is the reward function for the tax planner. Typically, the tax planner treats other agent as parts of the environment, then for the observation in timestep t, $o_p^t = \langle s^t, \mathbf{a}^t \rangle$ includes these general agents' joint state and action in the same timestep, while the action in timestep t, the tax planner receives its observation o_p^t from all general agents $a_p^t = \langle \boldsymbol{\theta}^t, \boldsymbol{\delta}^t \rangle$. Typically, at each timestep t, the tax planner receives its observation o_p^t from all general agents and outputs the action a_p^t , which contains all general agents' tax and allowance rates. Then it synchronously receives the reward r_p^t , the total rewards from all general agents, when other agents receive the shaped reward with tax and allowance. The policy of the tax planner is defined as $\pi_p \left(a_p^t \mid o_p^t\right)$. Therefore, the target of the tax planner is to maximize the following objective function:

$$\max_{\pi_p} J_p := \mathbb{E}_{\pi_p} \left[\sum_{t=0}^T r_p \left(o_p^t, a_p^t \right) \right].$$

In the training process, we use the approximated state action function $Q_p(o_p, a_p)$ to replace the cumulative reward, and the objective function then becomes:

$$\max_{\pi_p} J_p := \mathbb{E}_{\pi_p} \left[Q\left(o_p, a_p \right) \right].$$

Typically, the policy gradient-based optimization method is applied to train the tax planner. The gradient loss is therefore defined as follows:

$$\mathcal{L}(\phi_p) = \mathbb{E}_{\pi_p^{\phi_p}} \left[\nabla_{\pi_p^{\phi_p}} \log \pi_p \left(a_p^t \mid o_p^t \right) Q^{p, \pi_{\phi_p}^p} \left(o_p^t, a_p^t \right) \right],$$

where the tax planner's policy function parameters are represented by ϕ_p . Besides, as the tax planner has to maintain the balance on tax and allowance, the tax planner needs to minimize the following entropy in the learning process:

$$f(\pi_p) = \left| \sum_{t=0}^{T} \sum_{i=0}^{T} F_{\boldsymbol{\theta},\boldsymbol{\delta}}^i \left(o^t, \mathbf{a}_{-i}^t^*, a_i^t \right) \right|$$

As a result, the gradient loss $\mathcal{L}(\phi_p)$ can be denoted as:

$$\mathbb{E}_{\pi_p^{\phi_p}} \left[\nabla_{\pi_p^{\phi_p}} \log \pi_p \left(a_p^t \mid o_p^t \right) Q^{p, \pi_p^{\phi_p}} \left(o_p^t, a_p^t \right) \right] + \eta f \left(\pi_p^{\phi_p} \right), \tag{17}$$

where η is a hyperparameter weighting the entropy $f(\pi_p)$.

In light of the learning process of the tax planner, other general agents are trained within the approximated Optimal Pigovian Tax reward shaping as follows:

$$\mathcal{L}(\phi_i) = \mathbb{E}_{\pi_i^{\phi_i}} \left[\nabla_{\pi_i^{\phi_i}} \log \pi^i \left(a_i \mid s \right) \hat{Q}^{i, \pi_i^{\phi_i}}(s, \mathbf{a}) \right],$$
(18)

Algorithm 1 LOPT: Learning Optimal Pigovian Tax

1: Initialization: all general agents' policy parameters $\{\phi_i\}$, tax planner's policy parameters ϕ_p ;

- 2: for each iteration do
- 3: Generate a joint state-action trajectory with shaped rewards and tax/allowance rates as $\{\tau\}$;
- 4: **for** each state-action pair with shaped reward for each agent *i*, i.e., $\langle s_i, \mathbf{a}, r_i + F_i \rangle$ in $\{\tau\}$ **do**
- 5: Compute the new ϕ_i by gradient ascent on (18);
- 6: end for
- 7: **for** each tax planner state-action pair with global reward $\langle o_p, a_p, r_p \rangle$ in $\{\tau\}$ **do**
- 8: Compute the new $\hat{\phi}_p$ by gradient ascent on (17);
- 9: end for
- 10: $\phi_i \leftarrow \hat{\phi}_i, \phi_p \leftarrow \hat{\phi}_p$, for all $i \in \mathbb{N}$.
- 11: **end for**

where function $\hat{Q}^{i,\pi_i^{\phi_i}}(s,\mathbf{a})$ is defined as

 $r_i(s, \mathbf{a}) + F^i\left(s, \mathbf{a}^{-i^*}, a_i\right) + \gamma \max_{\mathbf{a}'} \hat{Q}^{i, \pi_i^{\phi_i}}\left(s', \mathbf{a}'\right).$

The typical learning process of LOPT is shown in Algorithm 1. In the next section, experiments in the Escape Room environment and the challenging Cleanup environment will show the performance of LOPT.

5 Experiment

5.1 Environments

In this paper, we have verified the performance of LOPT in two popular environments in self-interested multi-agent reinforcement learning:

i) Escape Room (ER) [37]: We first experiment on the Escape Room, ER(N, M) shown in Figure 4(a), where N > M. The target for N agents in this Escape Room environment is to successfully open the "door" by one or more agent(s) with the help of other M agents who pull the "lever" and then "escape". At each timestep t, there are 3 available states for each agent: door, lever, and start (the initial state). The action for each agent is to choose one of the 3 states as its new state, where the agent can keep its current state by selecting the same state as its action, and such behaviors receive a 0 reward. An agent receives a +10 reward if it moves to the door state and more than M other agents stay at the lever state. Otherwise, the reward for any movement which changes to a new state is -1. If any agent successfully opens the door, the episode will be terminated. If not, the episode ends after 5 timesteps, which is the max episode length. We build our LOPT method for the Escape Room environment based on the open-source implementation of [37], and conduct experiments on both N = 2, M = 1 and N = 3, M = 2 settings.

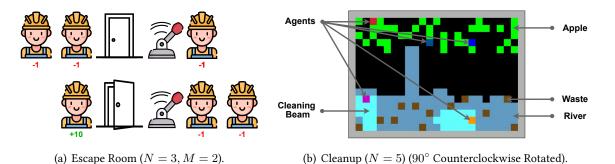


Figure 4: Simulated environments.

ii) *Cleanup* [10]: Furthermore, we conduct experiments on the Cleanup game with *N* agents shown in Figure 4(b). In the Cleanup environment [10], agents get +1 reward by collecting an apple, which spawns from the pre-defined apple spawn point in the map, and aim to collect apples from the field as more as possible. The apples are spawned at a variable rate, which decreases linearly from the parameter appleRespawnProbability as the aquifer fills up with waste over time at a certain rate defined by the parameter wasteSpawnProbability. So, if the levels of waste reach the parameter thresholdDepletion, no more apples or wastes will be spawned. If the waste density is no more than the parameter thresholdRestoration, the apple spawn probability restores to the parameter appleRespawnProbability. Otherwise, the apple spawn probability is:

$$p_{\text{appleSpawn}} = \frac{(1 - \text{wasteDensity}) \times \text{appleRespawnProbability}}{\text{thresholdDepletion} - \text{thresholdRestoration}}.$$
(19)

At each timestep t, agent i observes its surroundings as a normalized RGB image input o_t^i with a side length of view_size. Meanwhile, an agent additionally observes other agents' actions if they are visible to the agent. Then, an agent is able to move/rotate in the map, stay at its current position, and do nothing, or fire cleaning beams to clean wastes (the beam cannot penetrate wastes). Based on the open-source Cleanup implementation [31], we implement our LOPT method and run experiments with 3 different settings ranging from easy to hard: 1). N = 2 agents with a 7×7 map; 2). N = 2 agents with a 10×10 map; 3). N = 5 agents with an 18×25 map, where an additional "fire fining beam" action is equipped, and an agent can fine other agents by taking this action in this more challenging setting. As a result, an agent will receive a -1 reward for firing the fining beam, and the agent(s) hit by the fining beam will receive a -50 reward. But no agent will be penalized if the fining beam fails to hit any individual. In addition, to test our proposed LOPT with the fixed-orientated assumption in [37], we modify the N = 2 agents with a 10×10 map setting by setting the orientation of all agents as "up" and disabled rotation actions. Details for our 4 different experiment settings are shown in Table. 1.

Parameters	$N = 2, 7 \times 7$ map	$N = 2, 10 \times 10$ map	$N = 2, 10 \times 10$ map fixed orientations	$N=5,18\times25$ map
appleRespawnProbability	0.5	0.3	0.3	0.05
wasteSpawnProbability	0.5	0.5	0.5	0.5
thresholdDepletion	0.6	0.4	0.4	0.4
thresholdRestoration	0.0	0.0	0.0	0.0
rotationEnabled	1	1	×	\checkmark
view_size	4	7	7	7
max_steps	50	50	50	1000

Table 1: Experiment Settings for Cleanup Environment.

5.2 Implementations

We implemented the **LOPT** in both Escape Room and Cleanup environments. At each timestep t, the global observation o_{global}^{t} from the joint state s_t , and the joint action \mathbf{a}^{t} are fed to the tax planner as input. To better handle our challenging environments, we provide a "bank" variable to the tax planner to save rewards from taxes as available budgets for allowances, which supports the more sophisticated tax/allowance mechanism. Then, the current bank state o_{bank}^{t} and joint reward \mathbf{r}^{t} are also introduced to the observation:

$$o_p^t = \left\langle o_{global}^t, \mathbf{a}^t, o_{bank}^t, \mathbf{r}^t \right\rangle$$

The tax planner outputs the joint tax rate θ^t and the joint allowance rate δ^t . In addition, the tax planner outputs. Also, it outputs a percentage for rewards withdrawn from the bank as the budget ratio a_t^{bank} . So, the action for the current time step is:

$$a_t^p = \left\langle \boldsymbol{\theta}^t, \boldsymbol{\delta}^t, a_{bank}^t \right\rangle.$$

In addition, the entropy $f(\pi_p)$ is weighted by a hyperparameter η in Equation. 17 Concretely, in both environments with N agents, o_{bank}^t and \mathbf{a}^t are scalers, while \mathbf{a}^t , \mathbf{r}^t , $\boldsymbol{\theta}^t$ and $\boldsymbol{\delta}^t$ are N dimensional vectors. In the Escape Room games, the tax planner agent observes a multi-hot vector global states $o_{global}^t \in \{0, 1\}^d$ from the joint state s_t , where d = 3N. And in the Cleanup games, the global observation o_{global}^t is the global visual normalized RGB observation with the same width and height of the applied map.

In the Escape Room environment, the policy network for the tax planner is defined as follows: 1). a dense layer $h1_1$ of size 64 takes o_{global}^t as input and 3 dense layers $h1_i$, i = 2, 3, 4 of size 32 for \mathbf{a}^t , o_{bank}^t , and \mathbf{r}^t respectively; 2). the outputs of dense layers $h1_i$, i = 1, 2, 3, 4 are concatenated and fed to a dense layer h2 of size 32; 3). the output of dense layer h2 is fed to 3 dense layers $h3_i$, i = 1, 2, 3 of sizes 1, N, N and activation functions sigmoid, sigmoid, softmax, then output as a_t^{bank} , θ^t , δ^t respectively. While in the Cleanup environment, the policy network for the tax planner is defined as follows: 1). the global observation o_{global}^t is firstly fed to a convolutional layer conv1 of kernel size 3×3 , stride 1 and 6 filters; 2). the output of the convolutional layer conv1, \mathbf{a}^t , o_{bank}^t , and \mathbf{r}^t are fed to 4 two-layer dense layers $h2_i$, i = 1, 2, 3, 4 of size 32 and 32 respectively; 3). the outputs of dense layers $h2_i$, i = 1, 2, 3, 4 are concatenated and fed to an LSTM of cell size 128; 4). at last, the output of the LSTM is fed to the dense layers and output as a_t^{bank} , θ^t , δ^t respectively.

The settings of hyperparameters for baseline methods follow their previous work [10, 12, 37, 9]. For all experiments, the tuned hyperparameters of all baselines and LOPT are given in Table. 2-4, where: α is the learning rate; $\alpha_{schedule}$ is a list that contains the step and weight pairs for the learning rate scheduler; η is the weight for the entropy $f(\pi_p)$; ϵ in [37] decays linearly from ϵ_{start} to ϵ_{end} by ϵ_{div} episodes; β is coefficient for the entropy of the policy.

Hyperparameters			N	f = 2			N = 3						
	PG	PG-d	PG-c	LIO	LIO-dec	LOPT	PG	PG-d	PG-c	LIO	LIO-dec	LOPT	
α	1e-4	1e-4	1e-3	1e-4	1e-4	1e-3	1e-4	1e-4	1e-3	1e-4	1e-4	1e-3	
η	-	-	-	-	-	0.95	-	-	-	-	-	0.95	
$\epsilon_{\mathrm{start}}$	0.5	0.5	1.0	0.5	0.5	0.5	0.5	0.5	1.0	0.5	0.5	0.5	
ϵ_{end}	0.05	0.05	0.1	0.1	0.1	0.05	0.05	0.05	0.1	0.3	0.3	0.05	
$\epsilon_{ m div}$	100	100	1000	1000	1000	100	100	100	1000	1000	1000	100	
β	0.01	0.01	0.1	0.01	0.01	0.01	0.01	0.01	0.1	0.01	0.01	0.01	

Table 2: Hyperparameter Settings for Escape Room Environment.

L'Inne anno an anna atama		7×7 map							10×10 map									
Hyperparameters	AC	AC-d	AC-c	IA	LIO	PPO	MOA	SCM	LOPT	AC	AC-d	AC-c	IA	LIO	PPO	MOA	SCM	LOPT
α	1e-3	1e-4	1e-3	1e-3	1e-4	2.52e - 3	2.52e - 3	2.52e - 3	2.52e - 3	1e-3	1e-3	1e-3	1e-3	1e-4	1.26e - 3	1.26e - 3	1.26e - 3	2.52e-3
$\alpha_{schedule}$	-	-	-	-	-	[(5e5,	1.26e-3),	(2.5e6, 1.26	6e-4)]	-	-	-	-	-	[(le7, 1.26e-	4)]	[(5e5, 1.26e-3), (1e7, 1.26e-4)]
η	-	-	-	-	-	-	-	-	0.95	-	-	-	-	-	-	-	-	0.95
ϵ_{start}	0.5	0.5	0.5	0.5	0.5	-	-	-	-	0.5	0.5	1.0	0.5	0.5	-	-	-	-
ϵ_{end}	0.05	0.05	0.05	0.05	0.05	-	-	-	-	0.05	0.05	0.05	0.05	0.05	-	-	-	-
$\epsilon_{\rm div}$	100	100	100	1000	100	-	-	-	-	5000	1000	1000	5000	1000	-	-	-	-
β	0.1	0.1	0.1	0.1	0.1	1.76e - 3	1.76e - 3	1.76e - 3	1.76e - 3	0.01	0.01	0.1	0.01	0.01	1.76e - 3	1.76e - 3	1.76e - 3	1.76e - 3

Table 3: Hyperparameter Settings for Cleanup(N = 2) Environment.

Hyperparameters	PPO	MOA	SCM	LOPT
α	1.26e - 3	1.26e - 3	1.26e - 3	1.26e-3
$\alpha_{schedule}$	[(2e7, 1	.26e-4), (2e	e8, 1.26e-5)]	[(2.5e7, 1.26e-4)]
η	-	-	-	0.95
β	1.76e - 3	1.76e - 3	1.76e - 3	1.76e - 3

Table 4: Hyperparameter settings for Cleanup(N = 5).

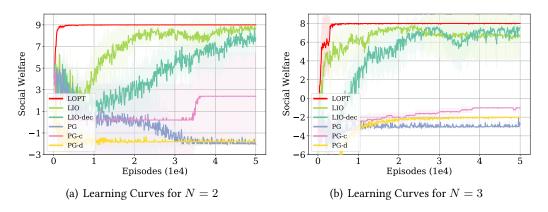


Figure 5: Results on Escape Room Environment.

5.3 Baselines

We compared the following baselines in our experiments: i). We first introduce three baselines of common reinforcement learning algorithms in previous works [10, 12, 37, 9], where Policy Gradient (**PG**) is applied in the Escape Room games and Actor Critic (**AC**) as well as Proximal Policy Optimization (**PPO**) are evaluated in the Cleanup environment; ii). We then compare state-of-the-art methods for dealing with social dilemmas, including **LIO** [37], **IA** [10], **MOA** [12], and **SCM** [9]. **LIO** learns to incentive other agents by giving one's own rewards to other agents so that cooperation emerges among agents and the social dilemma is alleviated. Meanwhile, a fully-decentralized LIO (**LIO-dec**) implementation [37] is also compared. Inequity Averse (**IA**) applies inequity-averse social preferences, which promotes solving social dilemmas by encouraging long-term cooperation [10]. Model of Other Agents (**MOA**) tries to solve social dilemmas by assessing causal influence on other agents via counterfactual reasoning [12]. Social Curiosity Module (**SCM**) combines the intrinsic rewards of curiosity and empowerment [9]; iii). In the Escape Room environment, **LIO** and **LIO-dec** are tested. Besides, agents who have extended discrete or continuous give-reward actions using Policy Gradient (**PG-d**, **PG-c**) [37] are also compared. In the Cleanup(N = 2) environment, **LIO**, **IA**, **MOA**, **SCM**, as well as extended Actor Critic methods (**AC-d**, **AC-c**) [37] are compared. In a more complex Cleanup(N = 5) environment, **MOA** and **SCM** are testified.

5.4 Results

Our experiments in both environments demonstrate that LOPT is able to solve the given social dilemmas and achieve better social welfare compared with all baselines. This arose in both ER and Cleanup environments because the tax planner in LOPT can reach near-optimal tax/allowance rate to model the Optimal Pigovian Tax reward shaping which internalize the externalitie. Also, LOPT results in fewer betrayals, which makes the learning process more stable. *Escape Room*: In ER(N = 2, M = 1) and ER(N = 3, M = 2), we observed from Figure 5(a) and Figure 5(b) that LOPT quickly converges to optimal values (8 and 9 for settings of N = 2 and N = 3 respectively).

The Optimal Pigovian Tax reward shaping from the tax planner is thought to help LOPT escape the given social dilemmas. PG Agents, optimized with policy gradient, strive to maximize their own rewards, leading to negative social welfare, resulting in a social dilemma. By augmenting the agents with discrete or continuous give-reward actions, the interplay of PC-d or PC-c agents may successfully open the "door" and get positive rewards. The results are still inconsistent, and these methods cannot properly solve the ER games. LIO agents and decentralized LIO-dec agents are able to "escape rooms" and reach a near-optimal state, respectively. However, the LIO and LIO-dec methods, while having more variance, are not stable and cannot prevent betrayals among agents from occurring, which results in fluctuations and failures to converge to the optimum.

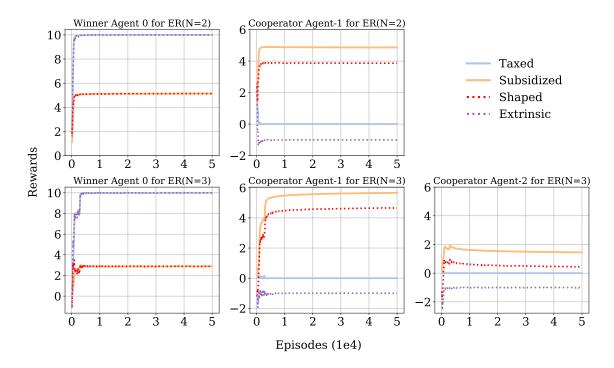


Figure 6: Rewards for Each Agent with Different Behaviors in Escape Room Environment. LOPT internalizes externalities and redistributes rewards among agents with taxes and allowances.

Additionally, by examining the behaviors of agents (shown in Figure 6), we demonstrate how the proposed LOPT internally reduces externalities while redistributing extrinsic rewards. Concretely, for the "Winner" agents who open the door and cause negative externalities, heavy taxes will be levied. On the contrary, the "Cooperator" agents who pull the lever and contribute to positive externalities will receive allowances. It is shown that the LOPT can provide accurate approximations of agents' externalities and then internalize them with Pigovian taxes and allowances by reward shaping.

Cleanup. From the learning curves in Figure 7, we observe that LOPT is able to reach better social welfare in all our settings. We then compare the proposed LOPT with PPO, SCM, and MOA baselines with scalability, in the more complex Cleanup(N = 5) scenario, where a larger map and a lower apple respawn rate are applied. Figure 7(d) shows that our proposed LOPT can scale to more complex scenarios and internalize the approximated externalities by learning Optimal Pigovian Tax reward shaping, which effectively helps agents to learn in social dilemma problems.

To illustrate how the proposed LOPT estimates externalities and affects the behaviors of agents, we explore the relationship between the behaviors and reward redistributions of agents. Figure 8(a) visualizes an example rollout with N = 5 agents, and it is evident that the proposed LOPT causes divisions of laborers (cleaner, harvester, and part-time) among agents by internalizing externalities so as to approach a better To find out how the LOPT internalizes the approximated externalities for each agent and causes divisions of laborers, the optimized tax/allowance mechanism is explored in Figure 8(c) for agents with different socially contributed behaviors: apple-collecting behaviors are taxed for causing negative externalities and social-good cleaning behaviors are allowanced for yielding positive externalities.

Also in Cleanup(N = 5) environment, we visualize and analyze the results from an example rollout in Figure 8. In addition, Figure 9(a), Figure 9(b), and Figure 9(c) show the relationship among the environmental states of the numbers of apples and wastes and the tax/allowance schemes given by the **LOPT**, where proper tax/allowance schemes are given for agents with different socially contributed behaviors. Figure 9(d) shows that the **LOPT** encourages agents to clean wastes efficiently and maintains the density of wastes at a relatively low level so that the apples are spawned at a relatively high rate.

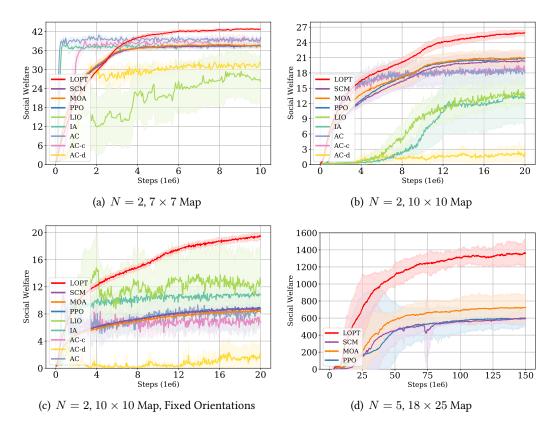
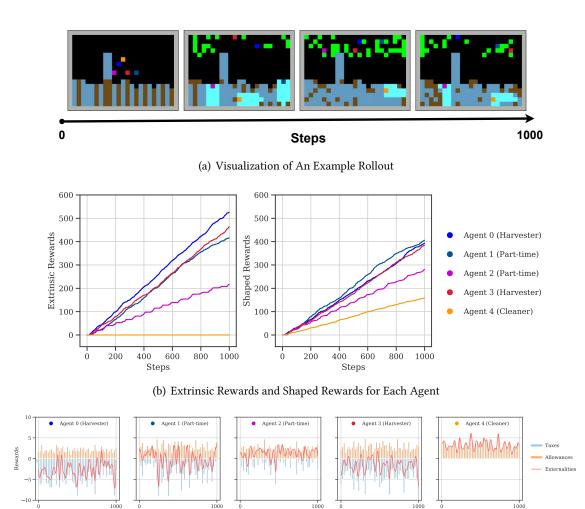


Figure 7: Results on Cleanup. (7(a), 7(b)) shows the learning curves for the proposed LOPT in Cleanup(N = 2); (7(c)) shows the learning curves for LOPT in Cleanup(N = 2) with the fixed-orientated assumption. (7(d)) scales to a more complex environment with N = 5 agents.



(c) Tax/Allowance schemes for Each Agent

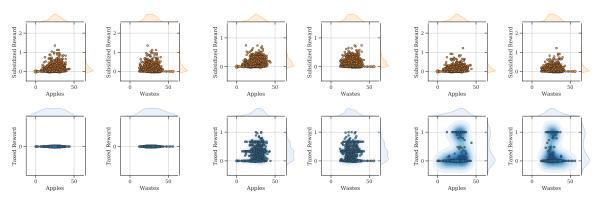
Steps

Sten

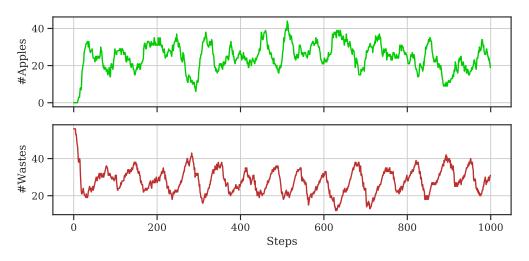
Steps

Sten

Figure 8: An Example Rollout for Cleanup(N = 5) Environment. (8(a)) visualizes this example rollout, where agents apply different social-good behaviors and divisions of laborers (cleaner, harvester, and parttime) emerge. (8(b)) shows the approximated Optimal Pigovian Tax reward shaping by the proposed LOPT. (8(c)) shows the reward shaping process of the LOPT in this episode, which demonstrates how the LOPT internalizes externalities for agents with different socially contributed behaviors.



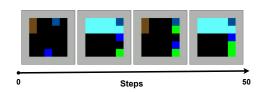
(a) Tax/Allowance Schemes with Envi- (b) Tax/Allowance Schemes with Envi- (c) Tax/Allowance Schemes with Environmental States for Cleaner Agents ronmental States for Harvester Agents ronmental States for Part-time Agents



(d) Number of Apples and Wastes in the Environment

Figure 9: An Example Rollout for Cleanup(N = 5) Environment, supplemental results for Figure 8. (9(a), 9(b), 9(c)) illustrate relationship of environmental states (the number of apples/wastes) and the tax/allowance schemes given by the **LOPT** for 3 types of agents with different socially contributed behaviors. (9(d)) shows the amount for apples and wastes during the episode.

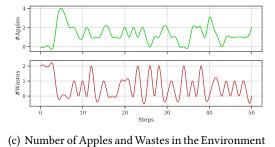
Furthermore, we provide visualized and analyzed results from example rollouts in Cleanup(N = 2) with both the 7×7 and the 10×10 maps. As illustrated in Figure 10-13, our proposed **LOPT** is able to internalize externalities in all of our Cleanup experiment settings and provide approximated Optimal Pigovian Tax reward shaping to greatly alleviate the social dilemmas.

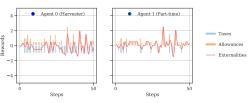


Agent 0 (Harvester) Agent 1 (Parttime) Agent 1 (Parttime)

(a) Visualization of The Example Rollout

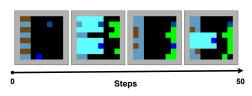
(b) Extrinsic Rewards and Shaped Rewards for Each Agent



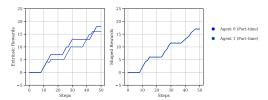


(d) Tax/Allowance Schemes for Each Agent

Figure 10: An Example Rollout for Cleanup(N = 2) Environment with A 7 \times 7 Map.



(a) Visualization of The Example Rollout



(b) Extrinsic Rewards and Shaped Rewards for Each Agent

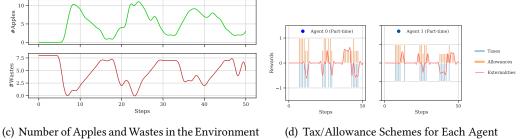
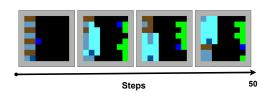
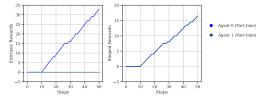


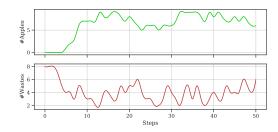
Figure 11: An Example Rollout for Cleanup(N=2) Environment with A 10×10 Map.





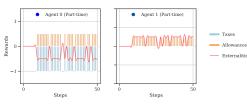
(a) Visualization of The Example Rollout

(b) Extrinsic Rewards and Shaped Rewards for Each Agent



(c) Number of Apples and Wastes in the Environment

Figure 12: Number of Apples and Wastes in the Environment



(a) Tax/Allowance Schemes for Each Agent

Figure 13: An Example Rollout for Cleanup(N = 2) Environment with A 10×10 Map and Fixed Orientations.

As a result, LOPT internalizes externalities and provides approximated Optimal Pigovian Tax reward shaping (shown in Figure 8(b)) to guide the agents to better results.

6 Conclusion

In this paper, the externality theory is first introduced to measure the influence of agents' behavior on social welfare. And based on the externality theory in the MARL area, the Learning Optimal Pigovian Tax method is proposed to deal with social dilemmas. A centralized agent, the "Tax Planner", is constructed to learn the tax/allowance allocation policy for each agent. Then through the Optimal Pigovian Tax reward shaping, each agent's externality is internalized, which encourages the agents to benefit the social welfare. Experiments have shown the superiority of the proposed mechanism for alleviating social dilemmas in MARL. In the future, we aim to build a decentralized Pigovian tax/allowance mechanism to learn reward shaping to internalize agents' externality with lower computation complexity.

References

- [1] G Christopher Archibald. Welfare economics, ethics, and essentialism. Economica, 26(104):316–327, 1959.
- [2] Tamer Başar and Geert Jan Olsder. Dynamic noncooperative game theory. SIAM, 1998.
- [3] Tobias Baumann, Thore Graepel, and John Shawe-Taylor. Adaptive mechanism design: Learning to promote cooperation. In *IJCNN*, 2020.
- [4] Mark Blaug. Welfare economics. In A Handbook of Cultural Economics, Second Edition. Edward Elgar Publishing, 2011.
- [5] Panayiotis Danassis, Zeki Doruk Erden, and Boi Faltings. Improved cooperation by exploiting a common signal. In AAMAS, 2021.
- [6] Heng Dong, Tonghan Wang, Jiayuan Liu, Chi Han, and Chongjie Zhang. Birds of a feather flock together: A close look at cooperation emergence via multi-agent rl. arXiv preprint arXiv:2104.11455, 2021.
- [7] Tom Eccles, Edward Hughes, János Kramár, Steven Wheelwright, and Joel Z Leibo. Learning reciprocity in complex sequential social dilemmas. arXiv preprint arXiv:1903.08082, 2019.
- [8] Jakob Foerster, Gregory Farquhar, Triantafyllos Afouras, Nantas Nardelli, and Shimon Whiteson. Counterfactual multi-agent policy gradients. In AAAI, 2018.
- HC Heemskerk. Social curiosity in deep multi-agent reinforcement learning. Master's thesis, Universiteit Utrecht Gerard Vreeswijk, 2020.
- [10] Edward Hughes, Joel Z Leibo, Matthew Phillips, Karl Tuyls, Edgar Dueñez-Guzman, Antonio García Castañeda, Iain Dunning, Tina Zhu, Kevin McKee, Raphael Koster, et al. Inequity aversion improves cooperation in intertemporal social dilemmas. In *NeurIPS*, 2018.
- [11] Aly Ibrahim, Anirudha Jitani, Daoud Piracha, and Doina Precup. Reward redistribution mechanisms in multiagent reinforcement learning. In Adaptive Learning Agents Workshop at AAMAS, 2020.
- [12] Natasha Jaques, Angeliki Lazaridou, Edward Hughes, Caglar Gulcehre, Pedro A Ortega, DJ Strouse, Joel Z Leibo, and Nando de Freitas. Intrinsic social motivation via causal influence in multi-agent RL. Arxiv preprint arXiv:1810.08647, 2018.
- [13] Jens Kober, J Andrew Bagnell, and Jan Peters. Reinforcement learning in robotics: A survey. The International Journal of Robotics Research, 32(11):1238–1274, 2013.
- [14] Peter Kollock. Social dilemmas: The anatomy of cooperation. Annual Review of Sociology, pages 183-214, 1998.
- [15] Raphael Koster, Dylan Hadfield-Menell, Gillian K. Hadfield, and Joel Z. Leibo. Silly rules improve the capacity of agents to learn stable enforcement and compliance behaviors. In AAMAS, 2020.
- [16] Guillaume Lample and Devendra Singh Chaplot. Playing fps games with deep reinforcement learning. In AAAI, 2017.
- [17] JZ Leibo, VF Zambaldi, M Lanctot, J Marecki, and T Graepel. Multi-agent reinforcement learning in sequential social dilemmas. In AAMAS, 2017.
- [18] Adam Lerer and Alexander Peysakhovich. Maintaining cooperation in complex social dilemmas using deep reinforcement learning. *arXiv preprint arXiv:1707.01068*, 2017.
- [19] Wenhao Li, Xiangfeng Wang, Bo Jin, Dijun Luo, and Hongyuan Zha. Structured cooperative reinforcement learning with time-varying composite action space. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(11):8618–8634, 2022.
- [20] Andreu Mas-Colell, Michael Dennis Whinston, and Jerry R Green. Microeconomic theory. Oxford University Press, 1995.
- [21] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A Rusu, Joel Veness, Marc G Bellemare, Alex Graves, Martin Riedmiller, Andreas K Fidjeland, Georg Ostrovski, et al. Human-level control through deep reinforcement learning. *Nature*, 518(7540):529–533, 2015.
- [22] Arthur Pigou. The economics of welfare. Routledge, 2017.

- [23] Alan Randall. Market solutions to externality problems: theory and practice. American Journal of Agricultural Economics, 54(2):175–183, 1972.
- [24] Anatol Rapoport. Prisoner's dilemma-recollections and observations. In Game Theory as a Theory of a Conflict Resolution, pages 17–34. Springer, 1974.
- [25] Tabish Rashid, Gregory Farquhar, Bei Peng, and Shimon Whiteson. Weighted QMIX: Expanding monotonic value function factorisation for deep multi-agent reinforcement learning. In *NeurIPS*, 2020.
- [26] Tabish Rashid, Mikayel Samvelyan, Christian Schroeder, Gregory Farquhar, Jakob Foerster, and Shimon Whiteson. QMIX: Monotonic value function factorisation for deep multi-agent reinforcement learning. In ICML, 2018.
- [27] Jun Sun, Gang Wang, Georgios B Giannakis, Qinmin Yang, and Zaiyue Yang. Finite-time analysis of decentralized temporal-difference learning with linear function approximation. In AISTATS, 2020.
- [28] Peter Sunehag, Guy Lever, Audrunas Gruslys, Wojciech Marian Czarnecki, Vinicius Zambaldi, Max Jaderberg, Marc Lanctot, Nicolas Sonnerat, Joel Z Leibo, Karl Tuyls, et al. Value-decomposition networks for cooperative multi-agent learning based on team reward. In AAMAS, 2018.
- [29] Ming Tan. Multi-agent reinforcement learning: Independent versus cooperative agents. In ICML, 1993.
- [30] Hanne van der Iest, Jacob Dijkstra, and Frans N Stokman. Not 'just the two of us': Third party externalities of social dilemmas. *Rationality and Society*, 23(3):347–370, 2011.
- [31] Eugene Vinitsky, Natasha Jaques, Joel Leibo, Antonio Castenada, and Edward Hughes. An open source implementation of sequential social dilemma games, 2019.
- [32] Eugene Vinitsky, Raphael Köster, John P Agapiou, Edgar Duéñez-Guzmán, Alexander Sasha Vezhnevets, and Joel Z Leibo. A learning agent that acquires social norms from public sanctions in decentralized multi-agent settings. arXiv preprint arXiv:2106.09012, 2021.
- [33] Oriol Vinyals, Igor Babuschkin, Wojciech M Czarnecki, Michaël Mathieu, Andrew Dudzik, Junyoung Chung, David H Choi, Richard Powell, Timo Ewalds, Petko Georgiev, et al. Grandmaster level in StarCraft II using multi-agent reinforcement learning. *Nature*, 575(7782):350–354, 2019.
- [34] Jane X Wang, Edward Hughes, Chrisantha Fernando, Wojciech M Czarnecki, Edgar A Duéñez-Guzmán, and Joel Z Leibo. Evolving intrinsic motivations for altruistic behavior. In AAMAS, 2019.
- [35] Woodrow Z. Wang, Mark Beliaev, Erdem Biyik, Daniel A. Lazar, Ramtin Pedarsani, and Dorsa Sadigh. Emergent prosociality in multi-agent games through gifting. In *IJCAI*, 2021.
- [36] Hua Wei, Nan Xu, Huichu Zhang, Guanjie Zheng, Xinshi Zang, Chacha Chen, Weinan Zhang, Yanmin Zhu, Kai Xu, and Zhenhui Li. Colight: Learning network-level cooperation for traffic signal control. In *CIKM*, 2019.
- [37] Jiachen Yang, Ang Li, Mehrdad Farajtabar, Peter Sunehag, Edward Hughes, and Hongyuan Zha. Learning to incentivize other learning agents. In *NeurIPS*, 2020.
- [38] Stephan Zheng, Alexander Trott, Sunil Srinivasa, David C Parkes, and Richard Socher. The ai economist: Taxation policy design via two-level deep multiagent reinforcement learning. *Science Advances*, 8(18):eabk2607, 2022.
- [39] Meixin Zhu, Xuesong Wang, and Yinhai Wang. Human-like autonomous car-following model with deep reinforcement learning. *Transportation Research Part C: Emerging Technologies*, 97:348–368, 2018.